HPC³ 2024
Problem A, English
Library of Wonders
Maximum Points: 5

The Library of Wonders is opening a new section, and they need your help to determine how they can organize it. The Library has k $(0 < k \leq 10^6)$ different types of books of equal dimensions, and N_i $(0 \leq i < k)$ books of the i-th type. Since The Library is meant to be wonderfully different on every visit, they want to find out how many distinct arrangements they can form.
However, the library has a tendency to preform cyclic shifts[1] on its sections. Therefore, they want you to find out how many different arrangements of books there are such that no arrangement can be made from any other by any number and combination of cyclic shifts.
All books of the same type are considered to be exactly equivalent.

[1]A cyclic shift is defined as a process that moves all books one space in the same direction, with books that leave the bounds of the arrangement wrapping around to the other side. Formally, if we represent an arrangement as an $n \times m$ matrix, a cyclic shift down will result in all (i, j) $(1 \leq i \leq n,\ 1 \leq j \leq m)$ being replaced by the item at $(i-1, j)$, and all $(1, j)$ being replaced by the item at (n, j).

Subproblem 1

The library has decided to make you construct a 2-dimensional arrangement of width W and height H $(0 < W \leq 10^6,\ 0 < H \leq 10^6)$.

They have also given you an array A of length k, where A_i is the number of books of the i-th type. It is guaranteed that the sum of A is equal to $W \times H$.

Input format

The first line of each input contains 3 integers W, H, and k.
The second line of each input contains k integers: The content of array A.

```
W  H  k
A[0]  A[1]  A[2]  …  A[k]
```

Output format

The first and only line of each output contains 1 integer C.

```
C
```

Where C is the number of possible arrangements that can be made.

Example Test Cases

Input 1

```
1 1 1
1
```

Output 1

```
1
```

A 1x1 grid has 1 possible arrangement. So, the program should return 1.

Input 2

```
6 1 3
1 2 3
```

Output 2

```
10
```

A 6x1 grid with 1, 2, and 3 books of different types has 10 possible arrangements. So, the program should return 10.

Input 3

```
3 3 2
3 6
```

Output 3

```
12
```

A 3x3 grid with 3 and 6 books of different types has 12 possible arrangements. So, the program should return 12.

Subproblem 2

Next, you will construct arrangements for the 3-dimensional section of library. You will be given the dimensions for the section: width W, height H, and length L ($0 < W \leq 10^6,\ 0 < H \leq 10^6,\ 0 < L \leq 10^6$).

Once again, you have an array A of length k, where A_i is the number of books of the i-th type. It is guaranteed that the sum of A is equal to $W \times H \times L$.

Input format

The first line of each input contains 4 integers W, H, L, and k.
The second line of each input contains k integers: The content of array A.

```
W  H  L  k
A[0]  A[1]  A[2]  …  A[k]
```

Output format

The first and only line of each output contains 1 integer C.

```
C
```

Where C is the number of possible arrangements that can be made.

Example Test Cases

Input 1

```
12  1  1  3
2  4  6
```

Output 1

```
1160
```

A 12x1x1 grid with 2, 4, and 6 books of different types has 1160 possible arrangements. So, the program should return 1160.

Input 2

```
2  3  3  2
6  12
```

Output 2

```
1044
```

A 2x3x3 grid with 6 and 12 books of different types has 1044 possible arrangements. So, the program should return 1044.

Input 3

```
72  60  96  4
17280  86400  120960  190080
```

Output 3

```
231490207
```

A 72x60x96 grid with 17280, 86400, 120960, and 190080 books of different types has 231490207 possible arrangements. So, the program should return 231490207.

Subproblem 3

Now, the library has allowed you to expand to their multidimensional section. You will now construct arrangements in the form of an N-th dimensional rectangular prism of axis lengths given by array S of length l $(1 \leq l \leq 10)$ where each element S_j $(0 < j \leq l, 0 < S_j \leq 10^3)$ is the length of an axis.

Once again, you have an array A of length k, where A_i is the number of books of the i-th type. It is guaranteed that the sum of A is equal to the product of the elements of S.

Input format

The first line of each input contains 2 integers l and k.
The second line of each input contains l integers: The content of array S.
The third line of each input contains k integers: The content of array A.

```
l  k
S[0]  S[1]  S[2]  …  S[l]
A[0]  A[1]  A[2]  …  A[k]
```

Output format

The first and only line of each output contains 1 integer C.

```
C
```

Where C is the number of possible arrangements that can be made.

Example Test Cases

Input 1

```
4 5
4 3 3 5
43 30 75 32
```

Output 1

```
82946004
```

A 4x3x3x5 grid with 43, 30, 75, and 32 books of different types has 82946004 possible arrangements. So, the program should return 82946004.

Input 2

```
7 9
11 9 7 4 9 15 19 13
1000000000 1500000000 800000000 1100000000 900000000 1250000000
950000000 1150000000 1000000000 3240527600
```

Output 2

```
925581285900
```

A grid with books of different types described above has 925581285900 possible arrangements. So, the program should return 925581285900.